Experiments with the Forbidden Regions of Open Periodic Structures: Application to Absorptive Filters

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Summary—**Experiments with open periodic structures are presented which show a large change in transmission at the transition from allowed to forbidden regions. At the boundaries of the forbidden regions, the radial distribution of the fields of a wave impressed upon the structure changes from one corresponding to slow-wave propagation to a field distribution corresponding to radiation. At these boundaries, the local field shapes do not change very much. However, at the boundary a large decrease in transmission through the structure is found, but with little change in input VSWR; this implies a change from real characteristic impedance to about the same value of radiation resistance. This behavior suggests applications to sharp cutoff absorptive filters.**

I. Introduction

OPEN PERIODIC waveguiding structures, such as the helix, are well known to most readers. Such structures are employed in many microwave devices, the most notable example being the traveling-wave tube. In this paper, we consider the nature of the transition from a propagating to a forbidden region on an open periodic structure, and show how this transition may be employed in a frequency-sensitive filter.

By "open" periodic waveguiding structures, we mean here structures having uniform periodicity in a desired direction of wave propagation which are open to infinity in the plane transverse to the desired direction of propagation. The condition of "openness," difficult to define in purely geometric terms, means that any energy radiated from the structure in a transverse direction will not be reflected back to the circuit but rather will be scattered to infinity.

Microwave filter art is filled with examples of periodic structures, including waffle irons, periodically-loaded coaxial lines, and leaky-wall waveguide mode filters. (For a review of these, see Torgow.[1]) The first two types of filters are closed structures whose input impedance becomes reactive in the stop band, resulting in reflection of the input power. Leaky waveguide filters are open in a sense; however, in those which are uniformly periodic, the periodicity does not appear to have been chosen to produce slow-wave propagation, but rather to reduce reflection of fundamental and harmonic power, for example, by causing partial reflections to cancel. In the present discussion, it is the openness of the structures and their dimensions relative to the wavelength which distinguish these structures from well-known periodic filter circuits. The openness and the dimensions chosen permit use of the forbidden region in filtering, and make these structures unique, so far as the authors are aware, in application to filtering. It is the use of the forbidden region which permits realization of frequency-dependent (rather than mode-dependent) filters which are well matched in both pass and stop bands.

Forbidden regions are discussed, with the usual analytical interpretation, in Section II. An earlier test of a "holey" ladder circuit demonstrating experimentally the existence of forbidden regions is recalled in Section III, where initial experiments with open periodic filter circuits in various environments are also presented. Finally, the experiments are summarized and some of the characteristics and developmental problems associated with these filters are described (Section IV).

II. Forbidden Regions in Open Structures

Let us consider wave propagation on open structures, *i.e.*, those structures which tend to support propagation at some frequencies along a z axis and are open along a radial coordinate r to infinity, at least for some range of the angular coordinate ϕ. In accordance with the definition given in Section I, any energy radiated from the structure will scatter outward and not be reflected back toward the z axis. The structures to be considered here are assumed to be uniformly periodic in the z coordinate; they may be periodic in ϕ and they need not be circularly symmetric.

The E and H fields of waves propagating along right cylindrical circuits have functional dependences for the region outside the structure of the form

$$K_n(\gamma r) \exp j(\omega t - \beta z - n\phi), \tag{1}$$

where

$$\gamma^2 = \beta^2 - k^2, \qquad k^2 = \omega^2 \mu \epsilon.$$

$K_n(\gamma r)$ is the nth order modified Bessel function. If the structure is uniform in the z coordinate, expressions such as (1) express the total fields for a given mode; if the circuit is periodic in z, the total fields can be expressed as a sum of terms like (1) with a summation over appropriate phase constants, β. For $\gamma^2 > 0$, one has $|\beta| > |k|$, and the fields decay at large r as $(\gamma r)^{-1/2} \exp(-\gamma r)$; this rate of decay is far more than sufficiently rapid to ensure the absence of radiation in the radial direction.[2] When the structure is excited at frequencies to produce

Manuscript received June 24, 1963; revised November 11, 1963.
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[1] E. N. Torgow, "Microwave filters," Polytechnic Res. and Dev. Electronics, Brooklyn, N. Y., PRD Reports No. 7; July–October, 1960; *Electro-Tech.*, vol. 67, pp. 90–96; April, 1961.

[2] A. Sommerfeld, "Partial Differential Equations in Physics," Academic Press, New York, N. Y., p. 188 ff.; 1949.

propagation with $\gamma^2>0$, the fields hug the structure. The condition $|\beta|>|k|$ means that $v_{\text{phase}}<c$ where $c=(\mu\epsilon)^{-1/2}$ is the velocity of light in the medium surrounding the structure. In other words, the structure propagates a slow wave. For a structure uniform in z, the regions $|\beta|>|k|$ and $|\beta|<|k|$ occur on a k-β plot as shown in Fig. 1. Regions marked A in Fig. 1 are regions of loss-free slow-wave propagation. Note that for a uniform structure no frequency can be found for which slow-wave propagation cannot occur. Thus uniform structures exhibit no frequency-dependent filtering property, at least from this general point of view.

If the other sign of γ^2 occurs, $\gamma^2<0$, the fields at large distance from the structure decay roughly as $r^{-1/2}$; the fields in this case do not hug the structure, and in an open structure radiation *may* occur. The detailed analytical answers to what does occur in the regions are just beginning to appear in the literature.[3-6] In these regions, identified by the letter F on Fig. 1, propagation does not occur on open structures. These regions will be called *forbidden regions* after Sensiper;[7] in contrast, the regions marked A on Fig. 1 will be termed *allowed regions*.

The propagation constants for periodic structures having uniform pitch p must be of the form

$$\beta_m = \beta_0 + 2\pi m/p, \qquad m = 0, \pm 1, \pm 2, \cdots$$

where m is the number of the axial spatial harmonic. (For some circuits there are modes for which not all spatial harmonics are present; hence certain values of m will not occur.) The requirement $\gamma^2>0$ for allowed propagation regions is equivalent to the condition $|\beta|>|k|$ and becomes, for open *periodic* structures,

$$|\beta_m| = |\beta_0 + 2\pi m/p| > |k|.$$

For each spatial harmonic which exists on the structure on a k-β plot, there will be a pair of straight lines originating at $k=0$, $\beta=2\pi m/p$, below which propagation is allowed and above which propagation is forbidden. The total field of a wave on the structure is made up of a sum of all of the spatial harmonics; hence, propagation is permitted only in the regions allowed for

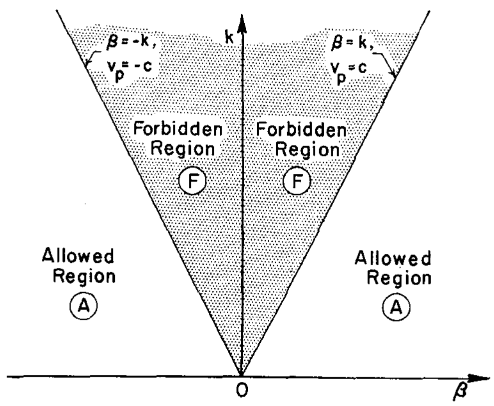

Fig. 1—Plot of k vs β showing regions where propagation on a uniform open structure is allowed (regions A) and forbidden (regions F). In allowed regions $|\beta|<|k|$; in forbidden regions $|\beta|>|k|$.

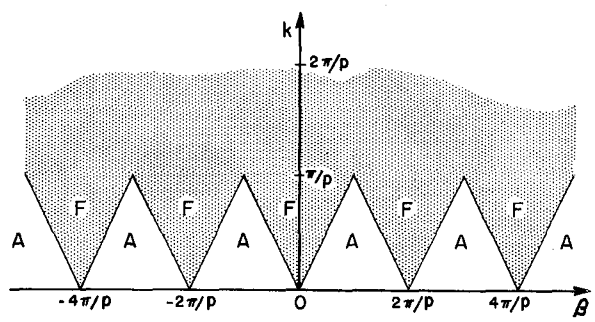

Fig. 2—Plot of k vs β diagram for open periodic structure. Regions A indicate where propagation is allowed and regions F where propagation is forbidden.

all spatial harmonics, that is, in the regions marked A in Fig. 2. Clearly, on such a structure, no propagation is allowed for frequencies such that $k>\pi/p$ corresponding to a free-space wavelength condition $\lambda/2<p$. The open periodic structure therefore exhibits a frequency-dependent filtering characteristic, in contrast with the uniform structure or the closed periodic structure. Experiments with these forbidden and allowed regions will now be described.

III. Experiments with Forbidden Regions

Results of several experiments are presented here to show the existence of the forbidden region and to show the properties of the cutoff obtained by crossing the boundary between allowed and forbidden regions on open periodic structures.

By way of introduction, let us first recall a test which showed clearly the existence of forbidden regions on a simple periodic structure.[8] A simple ladder circuit, as shown in Fig. 3, propagates in the z direction with the characteristic shown in Fig. 4. At very low frequencies, the circuit propagates with group and phase velocities

[3] A. Ishimaru and H. Tuan, "Frequency Scanning Antennas," Univ. of Washington, Dept. of Elec. Engrg., Seattle, Washington, Contract AF 19(604)-4098, ASTIA 265221, Tech. Rept. No. 54; April, 1961.

[4] A. Ishimaru and C. Hsieh, "Frequency Scanning of Slow Wave Antennas," Univ. of Washington, Dept. of Elec. Engrg., Seattle, Contract AF 19(604)-4098, ASTIA 270918, Tech. Rept. No. 57; August, 1961.

[5] P. W. Klock and R. Mittra, "On the Solution of the Brillouin (k-β) Diagram of the Helix, and its Application to Helical Antennas," 1963 PTGAP Internatl. Symp. on Space Telecommunications, Natl. Bur. Standards, Boulder, Colo.; July, 1963.

[6] A. Hessel and A. A. Oliner, "Mode Coupling Regions in the Dispersion Curves of Modulated Slow Wave Antennas," 1963 PTGAP Internat'l. Symp. on Space Telecommunications, Natl. Bur. Standards, Boulder, Colo.; July, 1963.

[7] S. Sensiper, "Electromagnetic wave propagation on helical structures," Proc. IRE; vol. 43, pp. 149–161; February, 1955.

[8] R. M. White, C. K. Birdsall, and R. W. Grow, "Multiple-ladder circuits for millimeter wavelength traveling-wave tubes," Proc. Symp. on Millimeter Waves, Polytechnic Inst. of Brooklyn, N. Y., pp. 376–377; 1959.

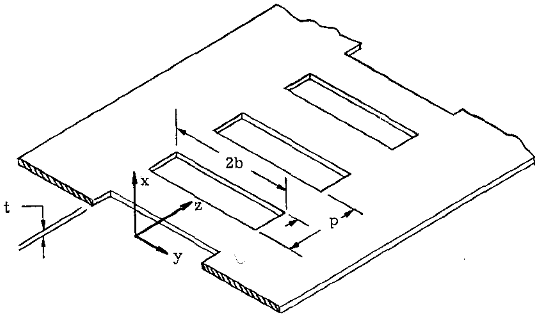

Fig. 3—The plane ladder slow-wave circuit.

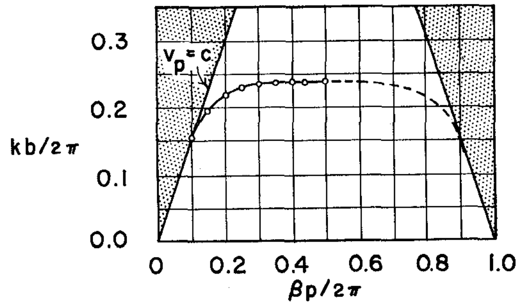

Fig. 4—Experimentally measured k-β plot for the plane ladder of Fig. 3. Solid curve is fundamental spatial harmonic, dashed curve is $m=-1$ spatial harmonic.

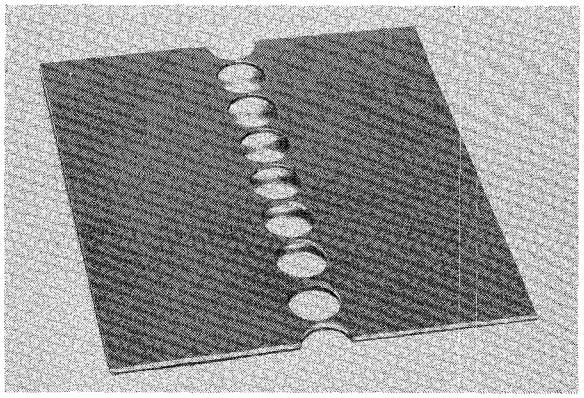

Fig. 5—Photograph of "holey" ladder circuit.

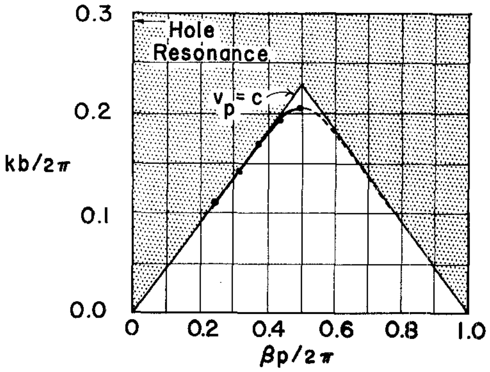

Fig. 6—ω-β curve for the ladder circuit of Fig. 5. For this holey circuit, the pitch-to-diameter ratio was 1.125. Cutoff occurred within the allowed region triangle, as shown, rather than at the circular waveguide TE_{11} mode cutoff which would occur at $kb/2\pi = 0.293$. (b is the hole radius.)

both equal to the velocity of light. As the frequency of slot resonance is approached, the fields become localized around the slots and the group velocity tends toward zero. The frequency of slot or obstacle resonance depends on the shape of the slot and, for rectangular slots, corresponds to a slot length $2b$ of about $\lambda_0/2$, or $kb \cong \pi/2$; thus $f_{slot} \cong c/4b$. As the slot shape is made more nearly square, the resonant frequency moves closer to the apex of the allowed region. What would happen if the resonant frequency were put *above* the apex and this fell in the forbidden region? Such a circuit (Fig. 5) was constructed by punching circular holes in a thin metal sheet and is called, simply, the holey circuit.

For the circuit of Fig. 5, one would expect the obstacle resonance to correspond to the TE_{11} circular waveguide cutoff, at a frequency factor $kb = 1.84$, or $kb/2\pi = 0.293$, where b is the radius of the hole. This resonance lies above the apex of the allowed region triangle which occurs at $kb/2\pi = 0.225$ for the ratio $p/2b = 1.125$ used in this test. Thus, if forbidden regions really exist, propagation cannot occur all the way up to the obstacle resonance frequency as it does with ladder circuits of other shapes. The observed propagation characteristic is shown in Fig. 6. At low frequencies the circuit propagates at the velocity of light along the boundary between allowed and forbidden regions; at higher frequencies, instead of propagating up to the hole (or obstacle) resonance, just within the allowed region a cutoff occurs at $\beta p = \pi$ (as would also occur in an enclosed structure). However, propagation at higher frequencies (which would occur on an enclosed structure) was *not* observed. This, to us, was a strikingly simple demonstration of the existence of allowed and forbidden regions of propagation.

The holey circuit, as well as many others, may be used in filters of the type proposed here. Indeed, using a very long holey circuit with a simple source, it was found that the signal decayed away from the launching end more and more rapidly as the frequency was raised above the highest resonance shown in Fig. 6. However, for early experimental work, the use of the well understood helix was preferable. Accordingly, a helix filter was designed and tested with the following constraints: the helix should exhibit a single propagating frequency region, and it should be physically enclosed to prevent radio interference, its "openness" being realized by the use of an absorptive liner within the outer conducting shell of the assembly.

The tape helix exhibits all spatial harmonics (both even and odd indices); therefore, the allowed-forbidden region structure is as sketched on Fig. 7. A typical helix used in a traveling-wave tube might have a phase velocity ratio $v_p/c = 0.045$ corresponding to interaction with 500-volt electrons; such a helix would have a multitude of operating frequency regions, as indicated by the

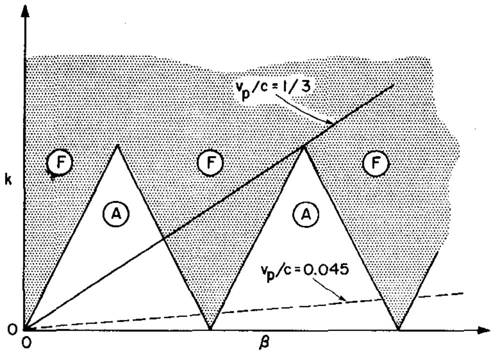

Fig. 7—k-β plot (idealized) for helix filter design. Dashed lines in allowed propagation regions (marked A) are approximate propagation characteristics for a few harmonics of 500 v helix ($v_p/c = 0.045$) as might be used in conventional traveling-wave tube. Velocity line for helix having a single mode of propagation is solid line corresponding to $v_p/c = 1/3$.

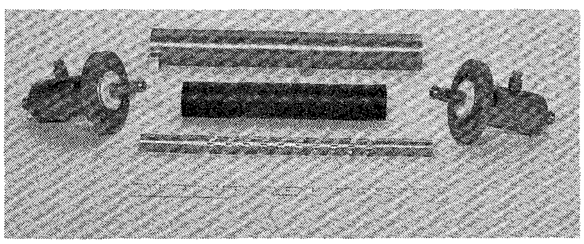

Fig. 8—Parts of helix assembly. Shown are: Type N coax to $1\frac{5}{8}$ inch coax transitions (on extreme right and left), helix, absorptive liner, and conducting barrel. Scale in inches.

dashed-line propagation characteristics sketched on Fig. 7.

In order to limit propagation to a single frequency region, it is necessary to have $v_p/c \geq 1/3$; this requirement results in a velocity line which just misses entering the second allowed region to the right of the origin (as shown in Fig. 7 by the solid line).[9] A helix was designed to have a velocity ratio of one third and to be fed directly from standard $1\frac{5}{8}$-inch OD coaxial transmission line (this line has inner and outer diameters 0.662 and 1.527 inches, respectively). Using these dimensions, straightforward design using $v_p/c = \sin \psi$ gives a helix having a pitch of 0.736 in, or 1.36 turns per inch, and a crossing of the boundary into the forbidden region at 3.84 kMc. The helix ID was chosen to be 0.500 in; the helix was ten pitches long. The helix and coaxial transitions are shown in Fig. 8, along with a barrel and absorber to be mentioned later.

Helix Actually Open

Initially the helix was fed by short sections of coaxial line on each end and tested with no nearby outer boundary. The helix and coaxial transitions were supported on a large polyfoam block away from reflecting surfaces. In this test, energy rejected by the structure might reflect back into the input line at the junction of the helix and the coaxial line or might radiate from the helix into the space (laboratory room) surrounding it. The insertion loss of the structure and the input VSWR were measured on this structure; data are plotted in Fig. 9. In considering these results it should be borne in mind that the transition from coaxial line to helix was quite abrupt; the outer conductors of the coaxial lines ended a short distance from the start of the helix which did not have a tapered pitch (as can be seen in Fig. 8). The most important feature of Fig. 9 is the large rise in insertion loss occurring between 3.80 and 3.90 kMc, a change in transmission of about 22 db. The calculated cutoff frequency—the transition from allowed to forbidden regions—was 3.84 kMc, as noted above.

The insertion loss in the pass band below 3.84 kMc is seen to be far too high for most filter applications. The 2 db reflection loss associated with the measured input VSWR (less than about 2.6:1 in the pass band) is too low to account for the measured pass band insertion loss. For this reason, it is felt that the large pass band loss resulted from radiation at the crude coax-to-helix transitions. (This loss is not an inherent feature of the helix transition, since traveling-wave tube work has shown that very low loss transitions can be made.)

Note that even 2 kMc above the transition into the forbidden region, the input VSWR is below 2.6:1. For absorptive filter applications, this rather good match may be highly desirable in those cases where good power transfer and little reflection are required. This behavior implies that the transition from propagation to radiation is accompanied by a change from real characteristic impedance to almost the same value of radiation resistance. Near the edge of the allowed region, where the helix fundamental mode group velocity goes to zero, the helix power-voltage impedance goes to infinity; in this region, no violent mismatching was observed although such a behavior might have been masked by circuit loss and/or the mismatch of the poor coax-to-waveguide transition.

Helix in Pipe of Small Diameter

A second series of tests was made with the ten-pitch-long helix enclosed in the barrel shown in Fig. 8. Test measurements were made with a bare pipe (to determine the effect of the pipe on the match at the input transition) and then with three different lossy liners.

With no absorptive liner and with a conducting pipe boundary, there are no forbidden regions and there can be fast-wave propagation. It was found that below 4 kMc, the insertion loss was reduced considerably from the values shown in Fig. 9, presumably because of the improvement in transition from input coaxial line to helix; essentially, zero loss was measured at several fre-

[9] This overlooks the existence of the h_{t1}' mode of Sensiper[7] (mode c, Fig. 4) that exists in the region $2\pi < \beta p < 4\pi$. The h_{t1}' mode is linearly independent of the h_{t0} or fundamental mode and hence can be avoided by proper excitation of the helix, a trick learned by traveling-wave tube builders. The authors thank the reviewer for this reminder.

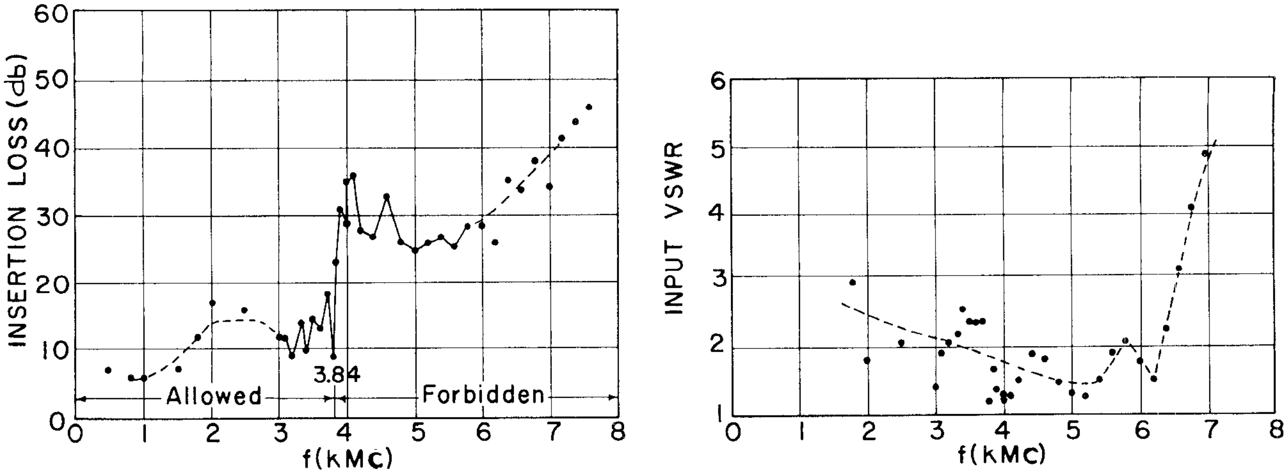

Fig. 9—Insertion loss and input VSWR vs frequency for helix supported on polyfoam block to simulate a completely open periodic structure. For this helix, the crossing of the asymptotic velocity line ($v_p/c=1/3$) occurs at 3.84 kMc as shown.

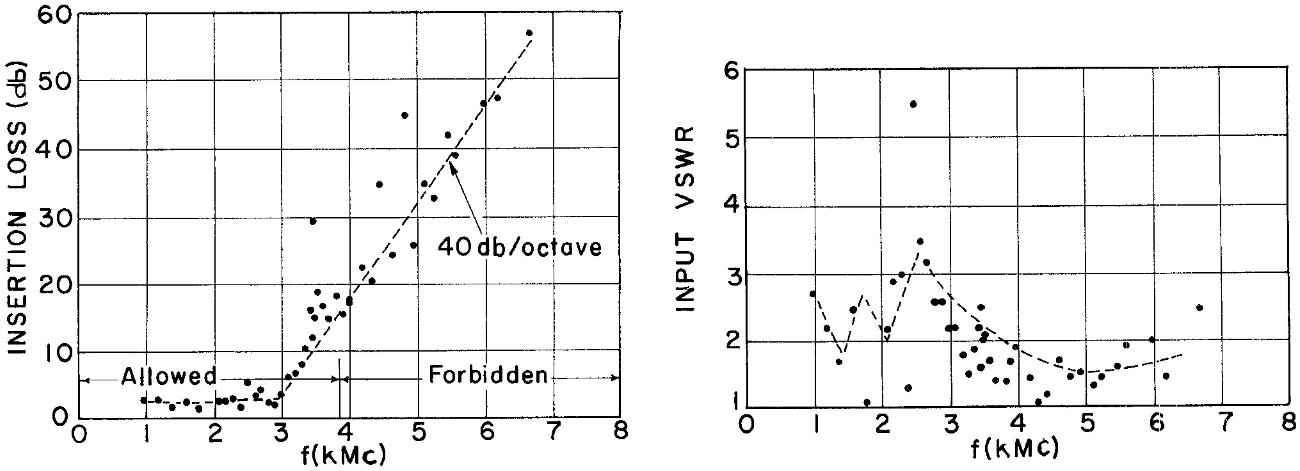

Fig. 10—Insertion loss and input VSWR vs frequency for ten-pitch helix in conducting pipe with relatively thick absorptive liner (1.090 inches ID, 1.500 inches OD, $9\frac{1}{8}$ inches long).

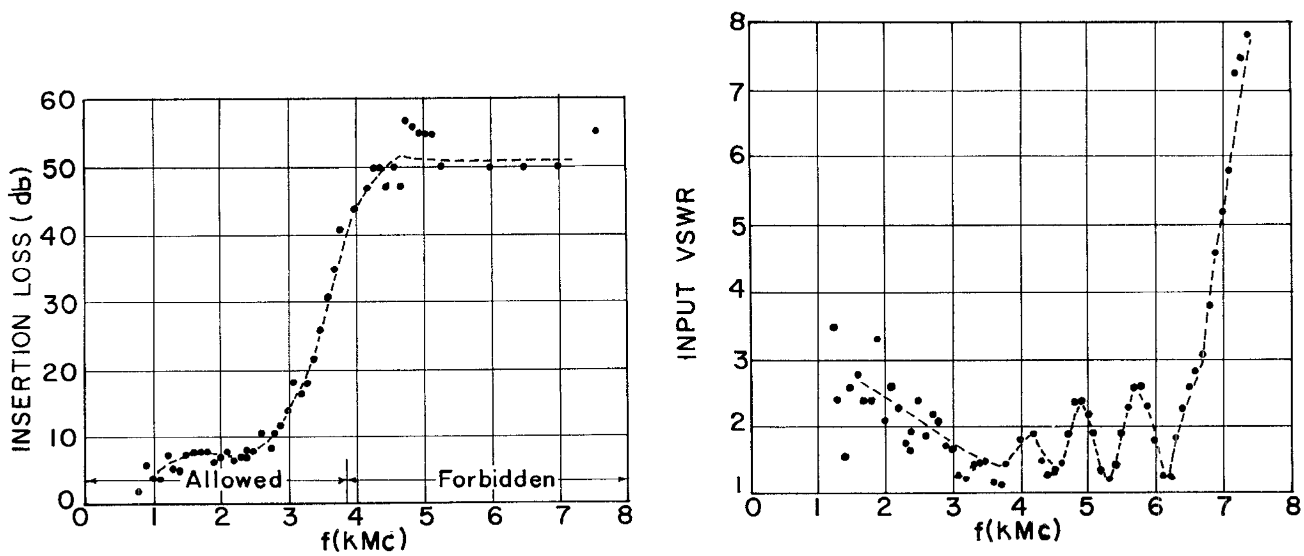

Fig. 11—Insertion loss and input VSWR vs frequency for ten-pitch helix in conducting pipe with absorptive liner identical in size with that of Fig. 10 but made of a different absorbing material.

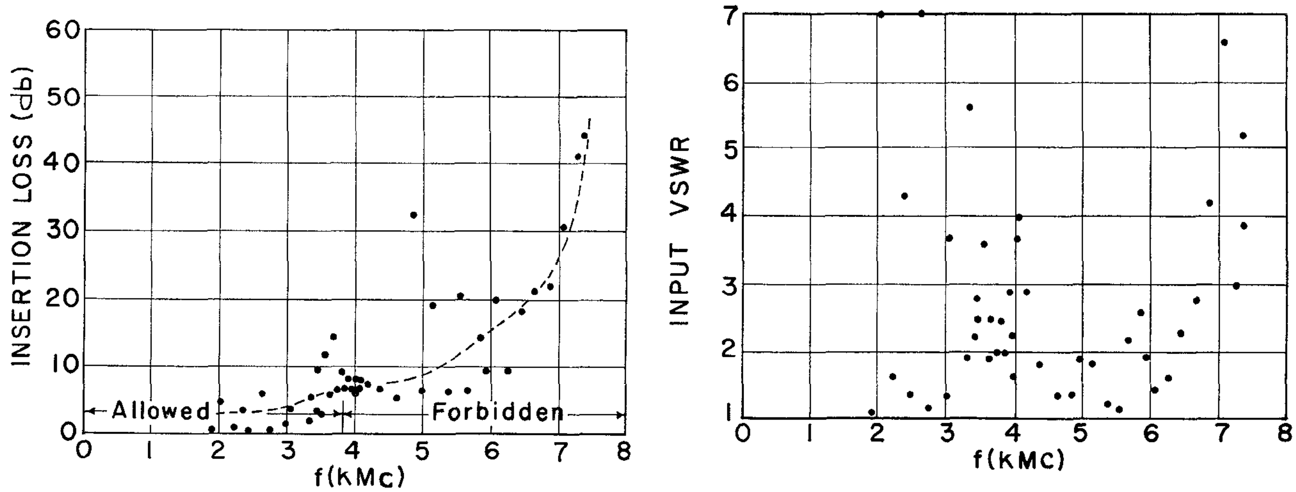

Fig. 12—Insertion loss and input VSWR vs frequency for ten-pitch helix in conducting pipe with relatively thin liner (1.265 inches ID, 1.500 inches OD, $9\frac{1}{8}$ inches long) made of same absorbing material as that of Fig. 10.

quencies and small losses were observed at other frequencies.

With an absorptive liner which does not reflect incident energy radiated from the helix, the forbidden regions should be present. When measured in one liner having an inner diameter of 1.090 inches and a length of $9\frac{1}{8}$ inches, the insertion loss curve for the ten pitch helix was as shown in Fig. 10. Pass band loss was generally less than 4 db; some of this loss was due to the proximity of the absorber to the helix itself and a fair fraction of the loss was accounted for by reflections as indicated by the measured input VSWR. The approach to cutoff with the small barrel and liner is more gradual than that for the open helix, beginning at about 3 kMc, in this case, and rising at approximately 40 db/octave. It is significant that in this and later tests, no evidence has been found of higher order modes of propagation occurring above the transition frequency for the lowest order mode, covering the range 1 to 11 kMc.

The cutoff and stop band properties of this structure depend critically upon the effectiveness of the absorber surrounding the helix. Fig. 11 shows the insertion loss for a filter containing an absorber of identical dimensions but of a different absorbing material. In this case, because of the different absorbing material, the cutoff is sharper than in Fig. 10 and the loss in the pass band is also somewhat higher. Fig. 12 shows test results obtained when the inside diameter of the absorptive liner of Fig. 10 was enlarged; the observed indistinct transition from allowed to forbidden regions of Fig. 12 suggests that in this case waves radiated from the helix are only slightly attenuated by the thin absorber, being reflected at the conducting metal barrel back onto the helix.

IV. Summary of Experiments and Discussion

The data given clearly shows experimentally that forbidden regions do exist on open periodic structures, and that the transition from an allowed to a forbidden region is accompanied by a large increase in the transmission loss of the structure. Test results have shown that the helical filter structure exhibits only one region of propagation in the region from 1 to 11 kMc; for this structure, the cutoff frequency was 3.84 kMc, so that at least second and third harmonics of the signals at frequencies as low as 2 kMc would be absorbed.

The experiments described were intended to demonstrate the use of the forbidden region in its application to filtering; in this sense, the experiments serve as an introduction to this topic. Only helix circuits have been tested; very crude transitions from the external coaxial transmission lines have been used. For applications in different frequency ranges, other open periodic circuits might be attractive. Although only low-pass structures have been described, it appears that it may also be possible to realize band-pass and reflective high-pass filters.

More refined development of these structures is certainly indicated, and it is clear that more attention should be given to transition design, absorber shape, location and choice of material, the question of the radiation pattern for energy leaving the periodic circuit, which is important in eliminating direct radiation from input through the filter to the output and for proper placement of the absorber, and designs for high-power operation.